Convolution-scattering model for staircase echoes at the temple of Kukulkan

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Chirped echoes from staircases at the temple of Kukulkan at Chichen Itza, Mexico have stimulated interest in archaeoacoustics since first reported by the author in 1998. In 2002, the author demonstrated chirped echoes and other acoustical effects at Chichen Itza tour following the First Pan-American/Iberian Meeting on Acoustics in Cancun, Mexico. Among those present was Nico Declercq. In 2004, Declercq et al claimed credit for the first scientific explanation for the chirp. That claim overlooks this author's earlier explanation at the Cancun Meeting. This paper suggests advantages of Lubman's earlier convolution-scattering approach to Declercq's later diffraction approach. Lubman models the clap-echo process as a time-invariant linear system. His result explicitly shows echo dependence on the incident sound spectrum. It is computationally far more efficient, provides instant auralization, and achieves higher near-field accuracy by avoiding the unnecessary assumptions of plane wave impingement on infinite corrugated periodic surfaces. It also yields more richly detailed echo sonograms. Echoes are calculated by convolving incident sound (e.g., handclaps) with staircase impulse responses. Impulse responses are modelled as scattering from staircase steps. Lubman's solution allows ethnologists, ethnomusicologists and others to conveniently simulate and auralize staircase echoes for *any* sound stimulus, including handclaps, voices, and ethnic sound instruments.

1 Introduction

Chirped echoes from staircases at the temple of Kukulkan at Chichen Itza, Mexico have stimulated much interest since first reported at scientific meetings by this writer in 1998 [1, 2, 3]. The temple – a world heritage site - has become an icon of Mexico.

Handclaps made while standing before the temple produce downward "chirped echoes". Hearing these echoes is, for most people, a remarkable listening experience. But echoes are an everyday experience. Why is this one so remarkable?

Fig.1 Temple of Kukulkan at Chichen Itza, Mexico. Photo shows one of its four 91-step staircases responsible for a remarkable echo - the subject of this paper.

This echo attracts notice because of an unexpected feature that defies listening experience.

Listeners expect echoes to sound like delayed replicas of their stimuli (a handclap, in this case). The chirped echo defies that expectation because its sound is so unlike the stimulus.

Most listeners react with astonishment. Some even impute magical or spiritual meanings to the echo.

The imputation of magical or spiritual meaning may not be so far fetched. Chichen Itza was a ceremonial city of the ancient Maya. Normally, only priests and caretakers were present there, except for special occasions. On those occasions thousands of Maya faithful probably filled the huge plaza to witness sacrifice and such spectacles as the spring equinox shadow that appears on the north-west balustrade of a chirping staircase. That ancient spectacle has become famous again, drawing thousands of tourists and pilgrims since its rediscovery in modern times.

The zigzag shadow is often described as the "descending feathered serpent". Some believe it represents the god Kukulkan's annual descent from heaven. The ancient Maya could also have heard the chirped echo at such times. It might have been stimulated by a priest, or by a synchronized chorus of hand clappers or percussion instruments. A synchronized clapping chorus make the chirp louder.

The chirped echo would have been especially meaningful to the ancient Maya, because it sounds amazingly like the primary call of the quetzal – a cloud forest bird widely venerated in Mesoamerica, but most famously by the ancient Maya who traded in its long and colourful tail feathers (coverts). Some believe the quetzal was a messenger of the gods. The descending serpent shadow may emulate the diving flight of the quetzal, which begins exactly at the spring equinox in the cloud forest. Thus, the ancient Maya would have had a special reason to hear the chirped echo as a magical or spiritual sound.

1.1 Objectives

This paper describes the chirped echo and compares it to the chirp of the quetzal

This paper outlines a mathematical explanation for the echo, first given at an acoustical meeting in Cancun, Mexico [1]. The explanation is useful not only for the physical insight it offers, but also for prescribing a practical method for analytically or numerically determining the echo response to any sound, whether live, recorded or mathematically described, such as handclaps and ethnic sound instruments.

The author’s formulation is compared with a later formulation of Declercq et al that makes use of diffraction theory [5]. Certain practical advantages of the author’s formulation are discussed.

2 Description of the chirped echo

A sonogram of the chirped echo is shown in Fig. 2. The sonogram depicts the chirp as graph of sound spectrum vs. time. Time, the abscissa, is marked in tenths of a second. The left ordinate is linear frequency, running from about 400 Hz to 3.1 KHz. Relative intensity in decibels [dB] is color coded as shown on the right axis, where red is the highest and black is the lowest intensity.

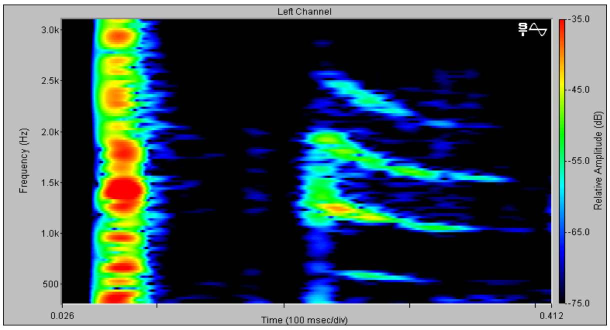

Fig. 2 Sonogram of chirped echo from the N. staircase of the temple of Kukulkan.

The colored vertical bar on the left of Fig 3 is the handclap stimulus. The chirped echo is comprised of the four curved lines on the right. It persists for less than 0.2 sec. The curved lines are harmonically related tones. Their downward curvatures tell that the chirp frequency falls with time.

3 Echo and quetzal chirp compared

The chirped echo seems even more astonishing when its sound and sonogram are compared with the quetzal [4]. A quetzal chirp sonogram is shown in Fig 3. Is the striking similarity with the chirped echo coincidental? Or did the ancient Maya intentionally design the echo to sound like the bird? If intentional, the chirped echo, created long before the Edison phonograph, is the world’s earliest known sound recording,.

4 Physical and mathematical models

Declercq et al provide an explanation for the chirped echo using diffraction theory in 2004 [5]. In it, the authors correctly observed that echoes depend strongly on the “type” of incident sound, but apparently could not determine that dependence.

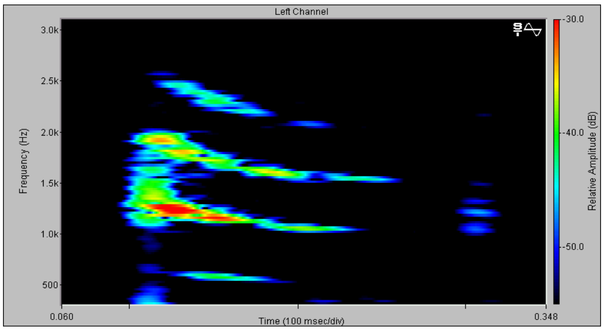

Fig. 3 Sonogram of a quetzal chirp bears a close resemblance to the chirped echo.

Declercq et al apparently overlooked this author’s earlier explanation at the First Pan-American/Iberian Meeting on Acoustics at Cancun, Mexico. Ironically that paper is cited in their own [5])..

That presentation explicitly showed the echo’s dependence on handclap spectrum (more generally, the time-frequency structure of impinging sound.)

It included a mathematical model of the chirped echo, echo simulation, auralization

The physical and mathematical model is outlined in Figs. 4 and 5. An observer’s handclap produces an acoustical disturbance, f(t) that propagates to and interacts with the staircase. Altered by the staircase, the disturbance returns to the observer’s ears as a chirped echo, g(t).

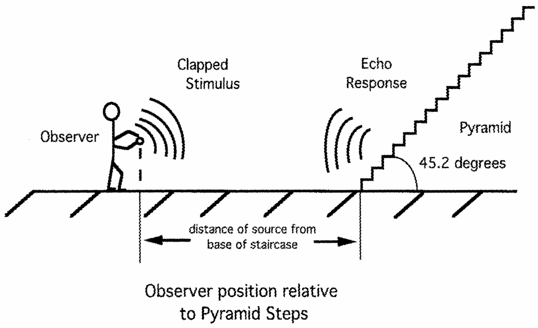

Fig. 4: Key physical elements of the chirped echo system reduced to two-dimensions

Any theory of the chirped echo must explain how an arbitrary stimulus such as a handclap f(t) is transformed into another sound, g(t), such as a chirped echo.

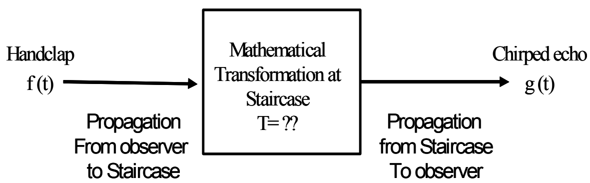

Figure 5: Schematic of the chirped echo system as a mathematical problem

4.1 Convolution-Scattering theory

The problem shown in Figs. 4 and 5 comprise a time-invariant linear system, as shown in Fig. 6. The solution is found simply by invoking the convolution theorem shown in Eq.(1). Here, $h(\tau)$ is the staircase impulse response.

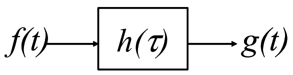

Fig. 6. Input-output relations for a linear system

$$g(t) = \int f(\tau) \cdot h(t - \tau)\, d\tau \qquad (1)$$

The convolution integral implies that the chirped echo g(t) is determined by the stimulus f(t) and the impulse response of the staircase, $h(\tau)$.

The staircase impulse response can be modelled as scattering from the staircase. For this purpose it is convenient to assume that the scattering centers are located at vertexes of stair risers and treads, as shown by the red dots in Fig. 7. In this two dimensional reduction each stair is treated as a point scatterer. Thus, the staircase impulse response is the time-ordered sum of scattered returns from the 91 stairs.

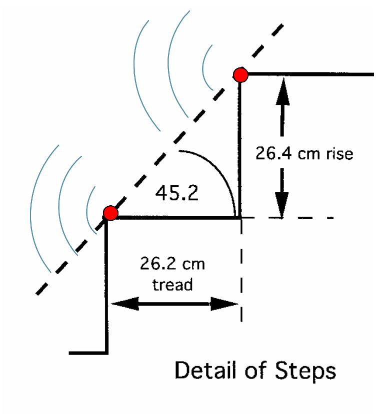

Fig. 7 Staircase modelled as an array of 91 point scatterers centred at intersections of risers and treads. Measured average dimensions are shown.

It follows from the convolution integral that the chirped echo can be defined in the frequency domain as well by Fourier transformations – this is another useful formulation.

$$G(j\omega) = F(j\omega).\ H(j\omega) \qquad (2)$$

where,

$$G(j\omega) = F\,[g(t)] \qquad (3)$$

$$H(j\omega) = F[h(t)] \qquad (4)$$

where,

F [y(t)] is the Fourier transform of y(t) (5)

The stimulus (handclap) power spectrum is

$$|F(j\omega).\ F^*(j\omega)| = |F(j\omega)|^2 \qquad (6)$$

where, * designates complex conjugate.

The echo power spectrum is

$$|G(\omega)|^2 = |F(j\omega)|^2\ .\ |H(j\omega)|^2 \qquad (7)$$

In words, the echo power spectrum is the product of the stimulus power spectrum (handclap) and the squared magnitude of the Fourier transformed impulse response.

The echo power spectrum is easily measured with standard instrumentation and measurement technique.

5 Convolution Results

Fig. 8 is the sonogram of a computer simulated chirped echo first shown at the 2002 Cancun meeting. The calculated echo was auralized as a *.wav.

This particular result was obtained by convolving a 1 ms sample of band-limited random noise with a mathematical model of the impulse response. The impulse response consisted of 91 values of time delay and 91 values of pressure amplitude. [The echo sound pressure varies as 1/2r, where 2r is the round trip distance between the clapper and the staircase scatterer]. Calculations for this example assumed the clapper stood 11 meters from the staircase.

Calculations took less than 1 second on an ordinary 1999 Dell desktop computer and the result - a *.wav file - was auralized immediately.

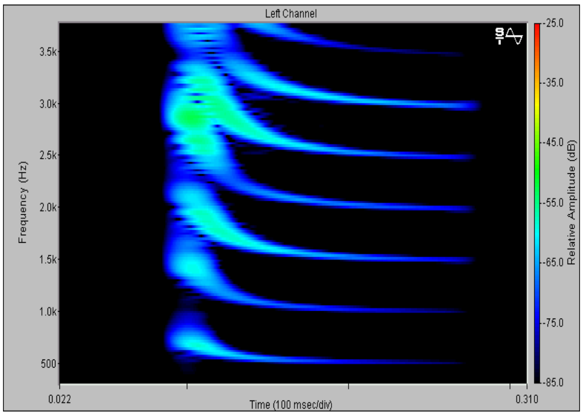

Fig. 8 Chirped echo simulated by convolution on a simple desktop computer.

In Fig 9, the sonogram of a chirped echo recorded at the same distance is shown for comparison.

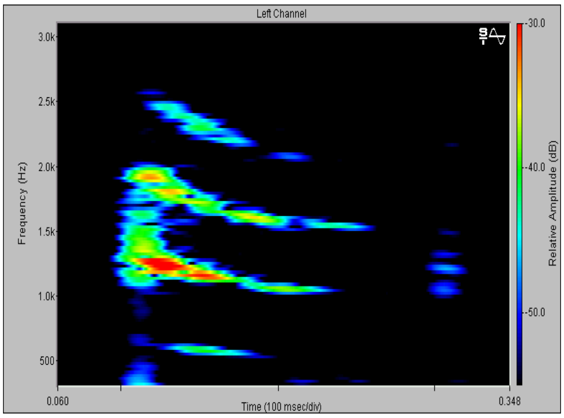

Fig. 9. Actual chirped echo from a handclap at the temple of Kukulkan's north staircase.

Several notable differences between Figs. 8 and 9 are observed.

The simulation of Fig. 8 is richer in harmonics than the actual echo recorded at the temple.

One reason is that no allowance was made for the handclap spectrum. Each handclap has a unique power spectrum. Some guides at Chichen Itza have learned to clap effectively to produce a strong echo.

It is simple to incorporate handclap spectra by suitable modelling f(t).

Another reason is staircase deterioration. In ancient times, staircases were periodically replastered with smooth and non-porous limestone plaster. Over time, the plaster eroded and the stairs become irregular and porous is due to weathering. Those features reduce the staircase scattering strength, especially at higher frequencies.

It is simple to incorporate the scattering strength of the staircase by modifying the staircase impulse response according to measurements of the staircase scattering strength. Doing so will require permission of the Mexican authorities (INAH).

It seems likely that the long stone staircases of every Mesoamerican pyramid chirped strongly when freshly plastered, and were even louder than they are today. If staircase riser heights and tread length dimensions are similar the chirped echo will sound like the quetzal. Today however, the chirped echoes are inaudible or barely audible at most Mesoamerican pyramids. However, the chirp can often be made audible by recruiting synchronized clapping choruses. A synchronized chorus of 20 tourists, for example, will increase the echo strength by 10 log (20) = 13 dB. With this formulation, the chirp of now - silent pyramids can be simulated from their physical dimensions.

6 Comparison of diffraction and convolution – scattering models

Declercq et al [5] employ a complex yet needlessly restrictive diffraction solution. For example, it models the staircase as a corrugated surface of infinite extent. In contrast, the convolution – scattering model puts no restriction on staircase extent.

Moreover, since Declercq et al's solution requires plane wave incidence on the staircase, it is valid only to the far field. The plane wave assumption leads to large errors at short ranges where the wave front curvature is significant. In contrast, the convolution - scattering solution gracefully accepts wave front curvature and is thus correct in both the near and far field.

Declercq et al's reported that their theoretical sonograms required huge amounts of time even using Ghent University's highest speed computer. Yet their graphic results seem far less impressive than the simulated echoe sonogram shown here. The simulation of Fig 9 took less than a second on an ageing desktop computer.

In their abstract, Declercq et al raised the "critical question" of what physical effects cause the chirped echo? The question could not be answered by their analysis because it is obscured by their diffraction formulation. But that question was answered earlier, easily and transparently in ref [2] with the convolution – scattering model.

7 Conclusions

The chirped echo phenomenon is amenable to many useful treatments, including diffraction, convolution scattering, and others not discussed here.

Among the merits of the convolution-scattering model are:

- Stimuli f(t) can be analytical, sound recordings, or live sounds
- Response echoes g(t) are efficiently calculated with ordinary desktop computers
- Calculated echoes are auralized (heard) instantly.
- Echo sonograms are readily obtained from g(t)
- Ethnographers and musicologists should find this tool useful in their attempts to reconstruct or imagine the sounds of rituals in the plaza.

This paper is an example of acoustical archaeology. With the publication of the monograph Archaeoacoustics [6] archaeologists now seem more willing to consider such previously ignored sounds as echoes around ancient monuments as true archaeological artefacts that may merit scholarly study.

Since many unusual sounds can arise by chance, the question of intent is central to archaeological acoustic studies. Nevertheless, present indications suggest that some ancient builders were good listeners and lived in a much quieter world. Some were capable of superb acoustical engineering beyond the previous imaginings of modern workers. They may have created sounds and soundscapes that were highly meaningful in their particular cultures. Archaeologists may be able to discover more about disappeared cultures by recreating the sounds of their natural and built environments.

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